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Dissipation in Nonlinear Shallow Water Waves

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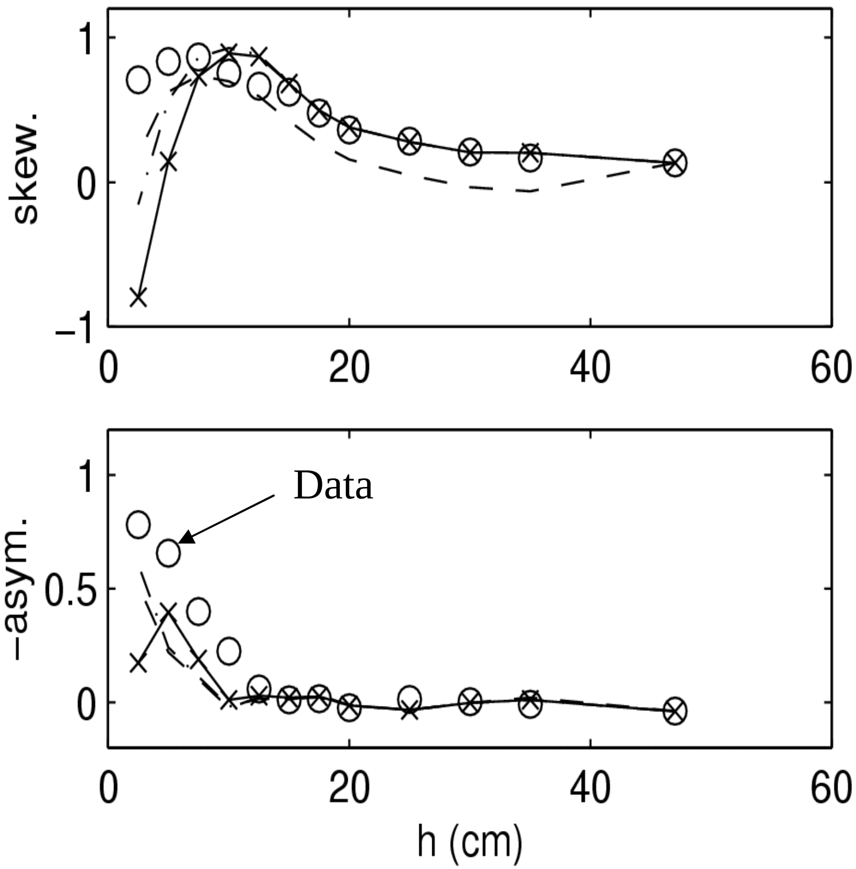
Background and Motivation

- Nearshore NOPP project – require nonlinear wave model to provide time series of velocity skewness for sediment transport
- In context of frequency domain models in NOPP: possible candidates include
  - Nonlinear mild-slope equation model (Agnon et al. 1993; Kaihatu and Kirby 1995)
  - Frequency domain extended Boussinesq model (Kaihatu and Kirby 1998)
- Bredmose et al. (2001, 2002): speed up calculations by computing nonlinear terms in *time domain*
- Frequency domain Boussinesq model has convenient time-domain analog – investigate concept further

Background and Motivation



Nonlinear frequency domain models vs. data of Mase and Kirby (1990)

- Dissipation in nonlinear frequency domain models : lumped parameter models (Battjes and Janssen 1978; Thornton and Guza 1983) with assumed frequency distribution
- Spectra comparisons usually reasonably good
- Third moment comparisons sometimes problematic, particularly in very shallow water
- Incorporation of local breaking descriptions would likely improve behavior

Background and Motivation

Generic time domain Boussinesq:

$$\eta_t + \left[(h+\eta)u\right]_x + O(kh)^2 = 0$$

$$u_t + g\eta_x + uu_x + O(kh)^2 = 0$$

Take Fourier transform, eliminate *u* between equations, combine:

$$A_{nx} + \frac{h_x}{4h}A_n - \frac{in^3k^3h^2}{6}A_n + \frac{3ink}{8h}\underbrace{\left(\sum_{l=1}^{n-1} A_l A_{n-l} + 2\sum_{l=1}^{N-n} A_l^* A_{n+l}\right)}_{\text{Results from nonlinearity; requires } O(N)^2 \text{ calculations}} = 0$$

Freilich and Guza 1984

Results from nonlinearity;
requires $O(N)^2$ calculations

Background and Motivation

Bredmose et al. (2001):

• Combine continuity and momentum equations by making use of first order substitutions and cast linear terms in frequency domain

• Take Fourier transform of individual terms of nonlinear products as an approximation:

$$\eta_x = \sum_{n=1}^{N} ik_n A_n e^{i(\int k_n dx - n\omega t)} \qquad u_x = \sum_{n=1}^{N} \frac{-igk_n^2}{\omega_n} A_n e^{i(\int k_n dx - n\omega t)}$$

• Inverse FFT these terms to get time series, multiply time series, sum, then FFT nonlinear sum – $O(N\log N)$ calculations

Extended Boussinesq Model

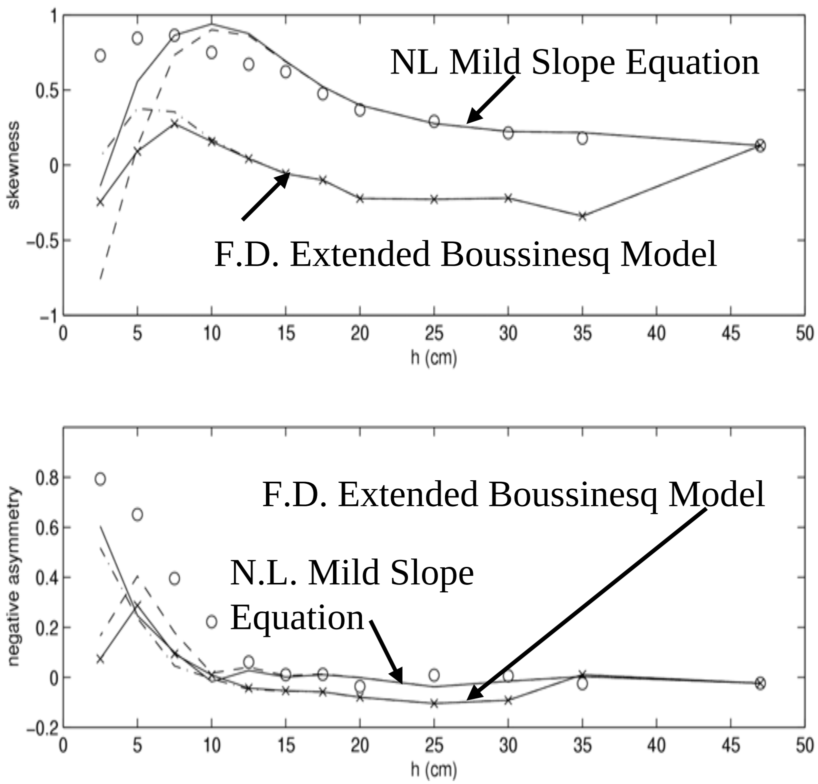
Extended Boussinesq model of Nwogu (1993) in 1D

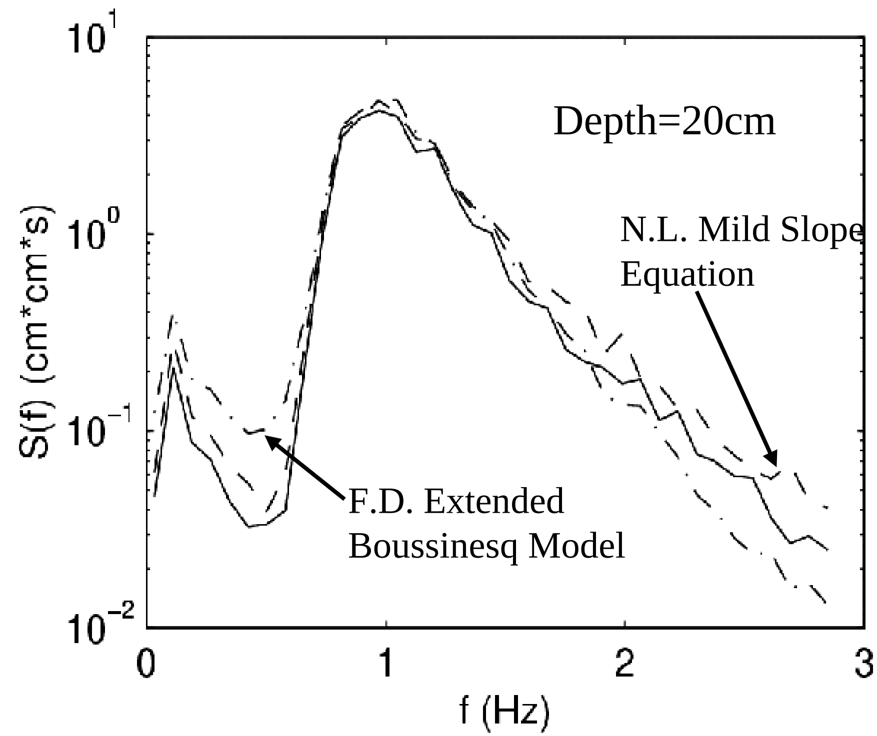
$$\eta_t + \left[(h+\eta)u_\alpha\right]_x + \left[\left(\frac{z_\alpha^2}{2} - \frac{h^2}{6}\right)hu_{\alpha_{xx}} + \left(z_\alpha + \frac{h}{2}\right)h\left(hu_\alpha\right)_{xx}\right]_x = 0$$

$$u_{\alpha_t} + g\eta_x + u_\alpha u_{\alpha_x} + z_\alpha\left(\frac{z_\alpha}{2}u_{\alpha_{xxt}} + \left(hu_{\alpha_t}\right)_{xx}\right) = 0$$

- Collapse to one equation for η difficult – must be done carefully to preserve advantageous dispersion and shoaling characteristics
- Kaihatu and Kirby (1998): parabolic frequency domain model – compared well to Berkhoff et al. (1982) and Whalin (1971)

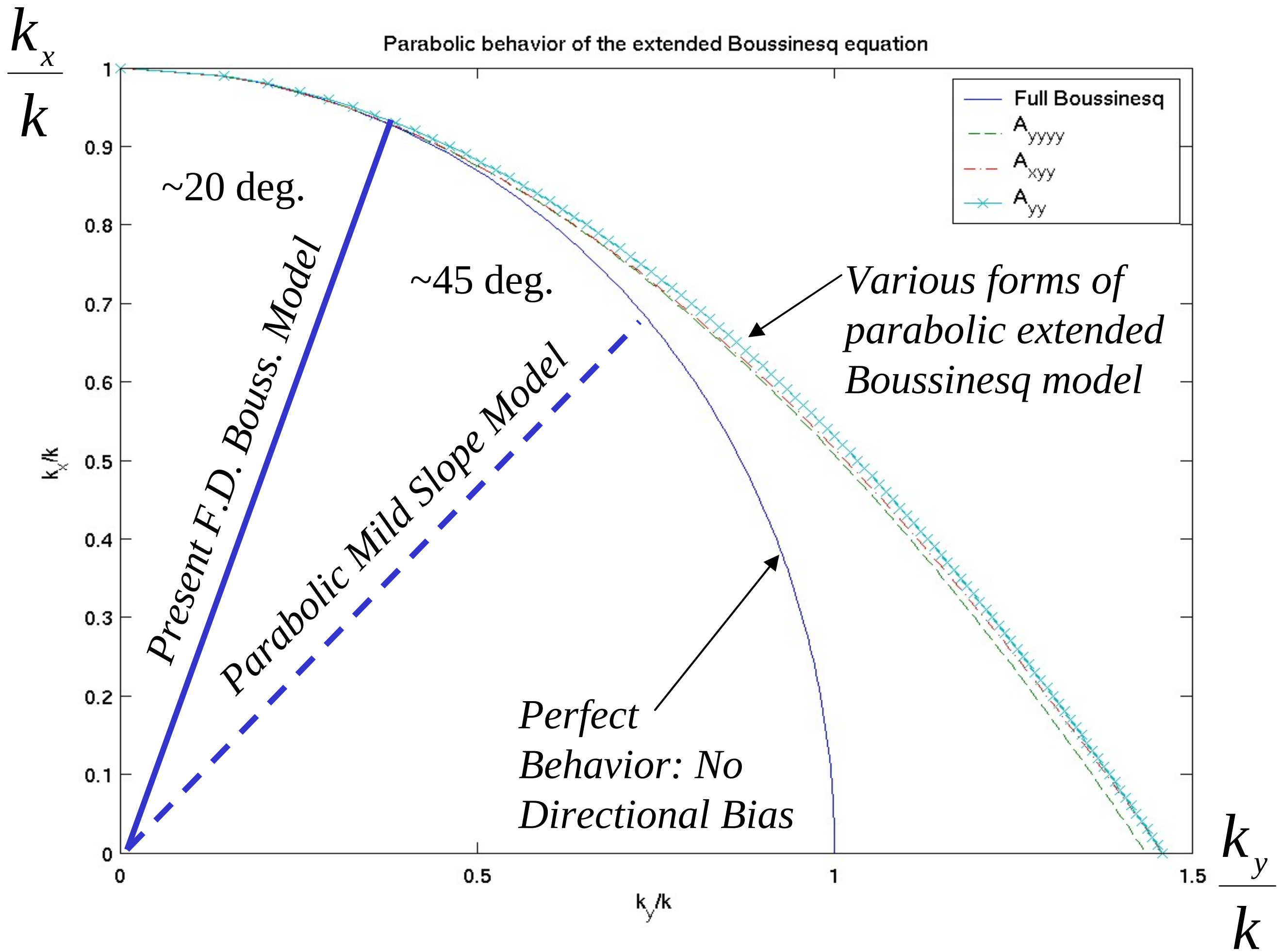
What the…? (1D Random Wave Shoaling)





Mase and Kirby (1992) Experiment – Higher Moments and Spectra

What the…? (Parabolic Model Behavior)



Now What?

- **NOTE:** The time domain extended Boussinesq equations do not suffer from these “issues”
- Likely cause: Collapsing equations into one for η - possibly much is getting lost in the maze of required substitutions (complexity of continuity equation)
- Alternative: Keep as two separate equations and solve for Fourier amplitudes of both η and u_α (A_n and B_n respectively)
- Advantage: Can add breaking model to momentum equation
- Approach of Bredmose – useful for incorporating more sophisticated transient breaking models (e.g. Veeramony and Svendsen 2000)

New Frequency Domain Model

Continuity Equation:

$$\left[h-3k_n^2h^2\left(\alpha+\frac{1}{3}\right)\right]B_{n_x}+\left[1-k_n^2h^2\left(\alpha+2\sqrt{1+2\alpha}\right)\right]h_xB_n$$
$$-3k_nh^3\left(\alpha+\frac{1}{3}\right)k_{nx}B_n+ik_nh\left[1-k_n^2h^2\left(\alpha+\frac{1}{3}\right)\right]B_n-i\omega_nA_n$$
$$+\frac{i}{4}\left\{\sum_{l=1}^{n-1}\Phi\left(k_{n-l},k_l\right)\left(A_lB_{n-l}+B_lA_{n-l}\right)e^{i\Theta(l,n-l,n)}+2\sum_{l=1}^{N-n}\Psi\left(k_{n+l},k_{-l}\right)\left(A_l^*B_{n+l}+B_l^*A_{n+l}\right)e^{i\Theta(n+l,-l,n)}\right\}=0$$

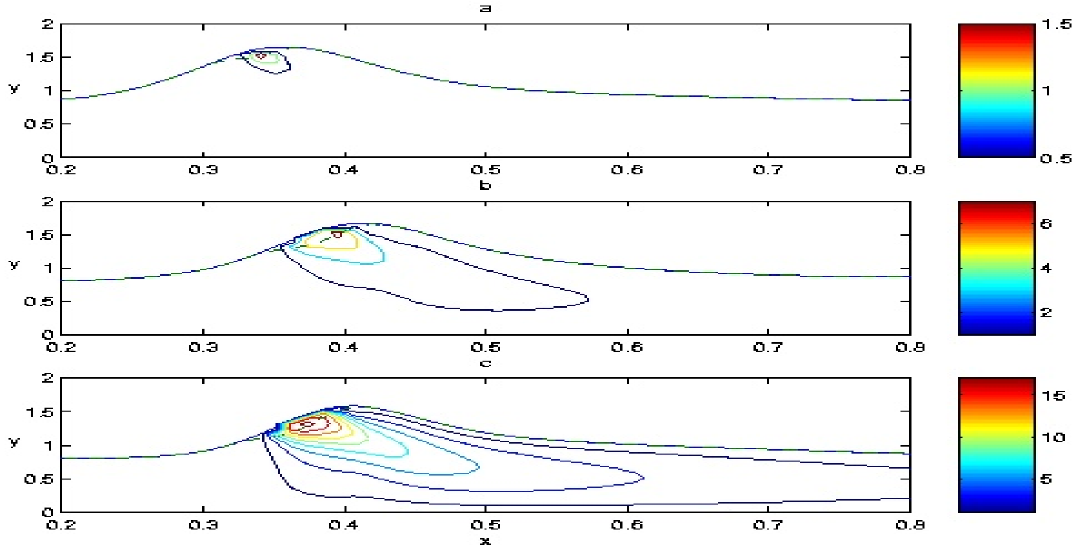
Momentum Equation

$$A_{n_x}+ik_n\left(1+\frac{2\omega_n^2\alpha h}{g}\right)A_n-\frac{2\omega_nk_nhh_x}{g}\left(1+\alpha-\sqrt{1+2\alpha}\right)B_n-\frac{i\omega_n}{g}\left(1-3\alpha k_n^2h^2\right)B_n$$
$$+\frac{\omega_n}{g}\alpha h^2k_{nx}B_n+\frac{i}{4g}\left(\sum_{l=1}^{n-1}\Phi\left(k_{n-l},k_l\right)B_lB_{n-l}e^{i\Theta(l,n-l,n)}+2\sum_{l=1}^{N-n}\Psi\left(k_{n+l},k_{-l}\right)B_l^*B_{n+l}e^{i\Theta(n+l,-l,n)}\right)=0$$

Dissipation

• Once we confirm model shows improvement in skewness, we can incorporate the Bredmose et al. (2001, 2002) spectral FFT technique

• Target transient dissipation model: Veeramony and Svendsen (2000)



Vorticity in a Breaking Wave – from model of Veeramony and Svendsen (2000)

Parameterizing High Frequency Behavior in Shallow Water

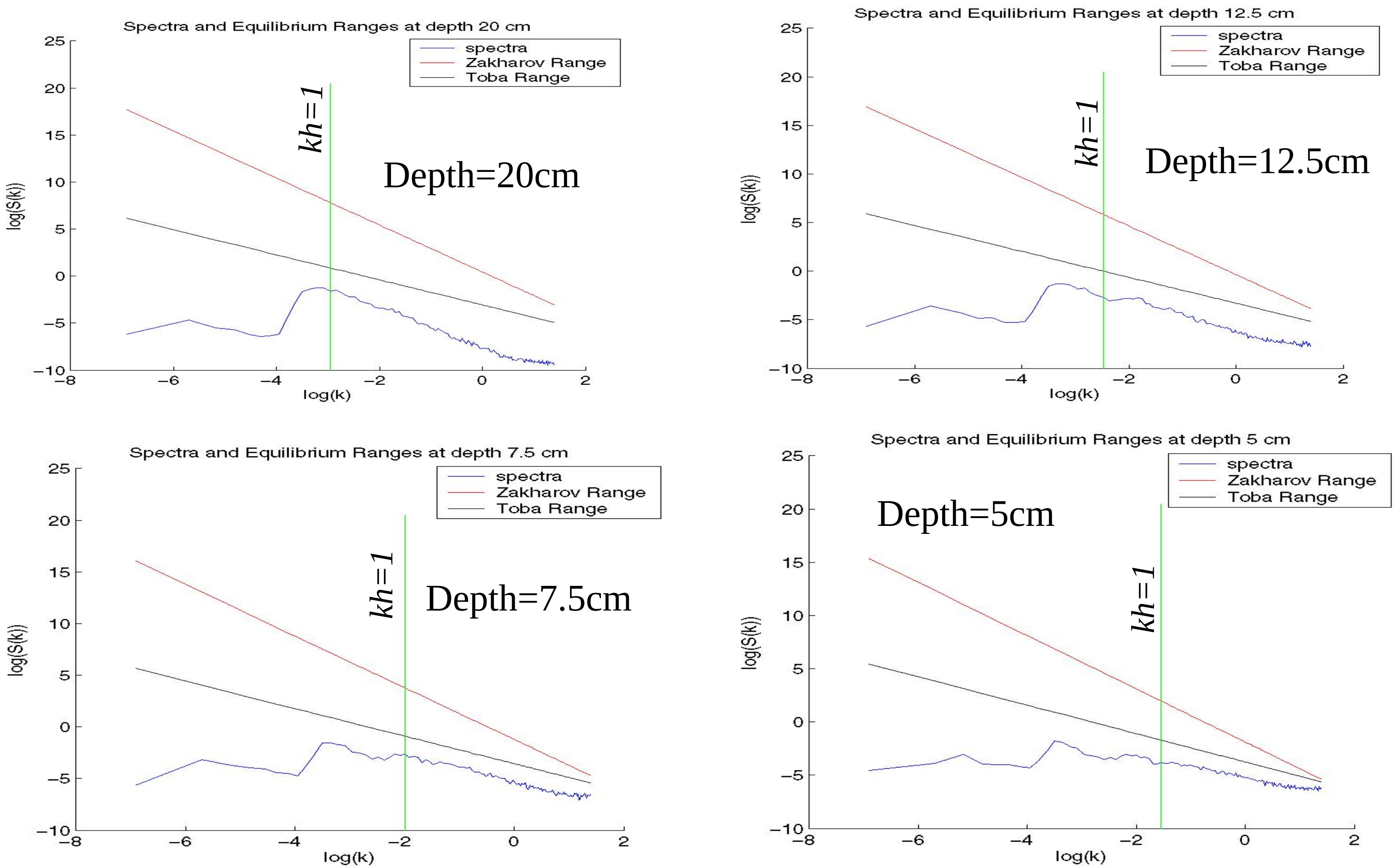
• Smith and Vincent (2002) developed equilibrium spectra descriptions for $f > f_p$ frequency range in the surf zone

• Based on extensive lab and field spectra, two ranges were developed:

$$S(k) = \beta_Z k^{-\frac{4}{3}} \longrightarrow$$ Zakharov range (kh<1): based on equilibrium spectra of solitons (Zakharov 1999)

$$S(k) = \beta_T k^{-\frac{5}{2}} \longrightarrow$$ Toba range (kh>1)

Parametric Form vs. Mase and Kirby (1992)



Further Work

- Demonstrate improved model performance over Kaihatu and Kirby (1998) (1D and 2D)
- Incorporate inverse FFT technique (Bredmose 2001, 2002)
- Incorporate dissipation model of Veeramony and Svendsen (2000)
- Compare model behavior to parameterized form of Smith and Vincent (2002) (*Mase and Kirby 1992 may be a singular data set due to relative water depth at wavemaker*)
- Can use to test NOPP model modularity – incorporation of "outside model"